

Indian Statistical Institute
First Semester Back Paper Examination 2004-2005
B.Math (Hons.) III Year
Introduction to Nonlinear Dynamics

Instructor : J Biswas

Max. Marks : 50

Attempt all questions :

1. Show that if a trajectory Γ is a periodic trajectory. [10]
2. Consider the system

$$\begin{aligned} \dot{x}_1 &= -x_2^3 \\ \dot{x}_2 &= x_1^3 \end{aligned}$$

Use the Liapunov function $V(X_1, X_2) = X_1^4 + x_2^4$ to prove that the origin is a stable equilibrium point of their system. Is it asymptotically stable? [10]

3. (a) Define the Poincare map.
(b) State the Poincare-Bendixson Theorem. [5+5]
4. (a) State Lienard's Theorem (for the system $x\ddot{x} + f(X)\dot{x} + g(x) = 0$).
(b) Let $F(x) = \int_0^x f(s)ds$ satisfy the hypotheses of Lienard's Theorem. Show that $\ddot{z} + F(\dot{z}) + z = 0$ has a unique stable limit cycle. [5+5]
5. (a) State the Poincare Hopf Index Theorem.
(b) Does there exist a nowhere vanishing smooth vectorfield on the sphere S^2 ? Same question for the torus T . Justify your answers. [5+5]